Oxbridge interview-like questions for training

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Abstract

The following document provides a series of author-made, interview-like questions for Oxbridge interview practice. The questions are composed solely by the author, and most of them, albeit not all, resemble typical Fermi problems. A Fermi problem involves estimating some parameter of interest from known quantities. The main goal is not precise calculation but rather evaluating the student's ability to reason through difficult concepts using simple physical intuition. Some of the problems include the author's own solutions, which represent one way of finding the quantity of interest. However, these solutions should serve only as guidelines and not as definitive methods for solving the questions. Each question must be adapted to each student according to what the interviewer aims to probe. For example, if a student is struggling with a particular concept in physics, the question could be redirected to something the student feels more comfortable with.

1. Fermi-like questions with solutions

In all of the Fermi-like questions it is very important to see the student's ability to **reason** physically in real-life situations and **approximate** the quantities of interest. So in these questions it's fine if a student (or an interviewer himself) makes some incredibly rough calculations, like, for example $\frac{\pi}{4} \approx 1$ or $\frac{e^2}{9} \approx 1$. So long as the student reasons correctly and gets roughly a correct order of magnitude it's good. (e.g. if we expect the order of magnitude to be $\approx 10^{22}$ there is no issue if the student has $\approx 10^{21}$)
In this section I provide several examples of how I would go at asking some of the questions myself with a more-or-less detailed thought process. This only represents my personal opinion and, naturally, can be subject to debating.

1.1 The banana question

How far up the mountain can I get with a banana? - one of the most classical Oxford questions made to confuse the student. Usually students don't understand what they're asked, and interviewer might need to ask a hint-question of 'Why do humans eat?'. The trick is to start thinking about the energy a single banana (rougly $E_{banana} \approx 100kcal \approx 4 \times 10^5 J$) gives to a human body and link it to a potential energy. Bearing in mind that not all the energy is converted into climbing up the mountain (a reasonable estimate, I believe, is the efficiency of $\eta \approx 0.3 - 0.4$). An example calculation:

$$\eta E = mg\Delta h - > \Delta h = \frac{\eta E}{mg} \approx \frac{0.4 \times 4 \times 10^5}{80 \times 10} \approx 200\text{m}. \tag{1}$$

One can then ask a bunch of leading-up questions which could include, e.g.:
(1) Include the kinetic energy calculations and compare it with potential (kinetic should be much, much less).
(2) If we could build a ladder all the way to the ISS, how many bananas would we need to eat to reach the ISS? (here, we can stress that potential energy is no longer approximated by a simple $E = mg\Delta h$ and one has to use the integral form $E = \int_{R_{earth}}^{R_{ISS}} G\frac{M_{earth} \times m}{r} dr$ and use it to check student's abilities to perform integration).

1.2 The Oxford balloon

Imagine I inflate the balloon in front of you. How many molecules of Nitrogen are trapped inside the balloon? Another Oxford classics, a bit more direct question though. Tests for a student's knowledge about the ideal gas law and some basics about the atmosphere. Here we only need student to remember the pressure of the atmosphere (or density, but if they don't know either it's fine to tell them) and % of N_2 in the atmosphere (roughly $\mu \approx 80\%$). The volume of the balloon can be approximated and the room temperature is around 300K. For example, by a sphere (any shape is reasonable, frankly speaking) of radius $R = 10$cm, giving the volume of $V = \frac{4}{3}\pi R^3$. The rough calculation goes as following:

$$PV = N_{air} k_B T - > N_{N_2} = \mu \frac{PV}{k_B T} \approx 0.8 \frac{10^5 \times \frac{4}{3} \times \pi \times 0.1^3}{10^{-23} \times 300} \approx 10^{23}\text{molecules}. \tag{2}$$

Where I did an extremely far-fetched approximation of $0.8 \times \frac{4 \times \pi}{9} \approx 1$. But, again, for an order of magnitude calculation this is fine. Potential further questions:
(1) Calculate a mass of all the electrons of all the N_2 molecules (a rather simple calculation but checks if the student remembers how many electrons does nitrogen have).
(2) Ask the student the assumptions behind the ideal gas equation. Next give the student a real gas equation

$$(P + \frac{a}{V_m^2})(V_m - b) = RT, \tag{3}$$

and, if the student doesn't know it, ask what might terms a (interaction between the molecules) and b (finite size of the molecules) represent, leading him back into the discussion about the assumptions of the ideal gas. If the student already knows the equation ask him to compute the number of molecules again, but with the real gas. Find constants a and b in the internet for nitrogen gas.

1.3 A seemingly random question

What is the mass of the hydrogen atoms lost on the sun due to a fusion each second? The question is designed to confuse a student and to ask an interviewer - 'what?'. Nonetheless, bear in mind a sheer number of assumptions which is coming. Start by asking a series of hinting questions. What is produced in fusion? (energy in the form of neutrinos/gamma-rays/other H atoms which hit other atoms and heat them up) Where does this energy end up first? (it is emitted in the form of light, for the purposes of this question let's assume 100% efficiency) Where does this light then end up? (at the Earth) Then, an interviewer can give a solar constant ($P_{sol} \approx 10^3 \frac{W}{m^2}$) and a distance from the Sun to the Earth ($R_{earth-sun}$ = 500 solar seconds $\approx 1.5 \times 10^{11}$m). We can then compute a surface of the sphere upon which the light is falling $S = 4 \times pi \times R^2_{earth-sun}$. From there, the energy emitted by the sun can be computed by $E_{sun} = S \times P_{sol}$. We also know that according to Einstein's mass-energy equivalence $E = mc^2$. Allowing us to compute the value as:

$$\Delta m = \frac{E_{sun}}{c^2} \times m_p = \frac{4 \times \pi \ (1.5 \times 10^{11})^2 \times 10^3}{(3 \times 10^8)^2} \approx 4 \times 10^9 kg. \tag{4}$$

Which turns out to be a very reasonable approximation. A student may further be asked, for example:
(1) In what distribution does the sun emit the light? (blackbody) (2) Estimate the temperature of the sun surface. (Can use the Wien's displacement law, which, if one wants to make a challenge, can be derived from the Plank's radiation law)

2. Fermi-like questions without solutions

In this section I provide a list of questions I think are interesting, with a minimal description.

2.1 A grain of sands in a bucket

Imagine I'm at a beach. How many grain of sands I can put in the bucket in my hand? - estimate the size of a single grain, estimate the volume of a bucket, divide one by another. Followup question- 'now, estimate number of grains of sands on all beaches in the entire world?'.

2.2 The original Fermi question

How many piano tuners are there in New York? - estimate the New York population, estimate number of pianos, estimate number of pianos a single person can tune. Followup - 'Why do pianos have to be tuned?'. Opens a discussion about harmonics and checking if students know about standing waves.

2.3 The weight of the atmosphere on my shoulders

Estimate the weight of the Earth's atmosphere - knowing pressure, we can easily find the weight by finding the surface area of the earth. Followup - 'How much carbon dioxide is in the atmosphere?'. Opens the discussion about how exactly does carbon dioxide trap the IR spectrum of the radiation.

2.4 The (not so) simple harmonic oscillator

Write me a force balance of a simple pendulum - this question is designed to test the differentiation skills of the student. We start from a simple force balance then proceed to a second-order differential equation. The solution to which can be hinted and asked to be checked. After that a damping term can be introduced for an equation to become more complex.

2.5 Where is my cup of tea?

Can I make a cup of tea on top the the Everest? - Ask what happens with height in the atmosphere, lead to reduction in the boiling point. Introduce a Clausius-Clapeyron equation. Define the 'ideal tea-brewing temperature' (it can be found in google). If the boiling temperature at the top of the Everest is lower than the ideal tea-brewing temperature then no, it's impossible to make a cup of tea.