Challenge 400: Divisibility Delights

Investigate these rules for divisibility!

Dr Reasoner is very keen on rules for checking the divisibility of large numbers.

One of her favourites is the test for divisibility by 11: form the digits of your "large" number into a calculation using just single digits by inserting + and - alternately, then evaluate the answer to the calculation and if this is divisible by 11, so is the original "large" number. For example, to check if 1, 234, 670, 775, 921 is a multiple of 11, find

1 - 2 + 3 - 4 + 6 - 7 + 0 - 7 + 7 - 5 + 9 - 2 + 1 = 0

0 = 0 x 11, so 1, 234, 670, 775, 921 **is** a multiple of 11.

(i) Explain **why** this rule works

Dr Reasoner had always thought that there was no simple and helpful check for divisibility by 7. (The ones you can find on the internet still require a lot of dividing by 7...)

However, Dr Reasoner's assistant, Phil, believes he has found another way to check for divisibility by 7. This is his rule:

When we write numbers in the usual English format, if the last three digits of the number(as one 3-digit number) subtract the second last three digits, plus the third last three digits, subtract the fourth last three digits… and so on, is divisible by 7, then the original number is divisible by 7.

For example, the number 1,234,567,891,011 is divisible by 7, because 011-891+567-234+001 = -546, which gives -78 when divided by 7. (the 1 at the front is seen as 001, to make up to three digits.)

(ii) Either prove that Phil's rule is correct, or show that the rule does not work.