United Kingdom
Mathematics Trust

Intermediate Mathematical Challenge

Wednesday 1 February 2023

Organised by the United Kingdom Mathematics Trust

England & Wales: Year 11 or below
Scotland: S4 or below
Northern Ireland: Year 12 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   Each incorrect answer to Questions 16-20 loses 1 mark;
   Each incorrect answer to Questions 21-25 loses 2 marks.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Intermediate Mathematical Challenge should be sent to:

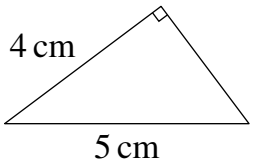
challenges@ukmt.org.uk

www.ukmt.org.uk

1. Which of these numbers is neither a multiple of 3, nor a multiple of 4?

A 16 B 21 C 28 D 34 E 45

2. What is the area of this triangle?

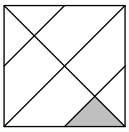


A $6\,\text{cm}^2$ B $7.5\,\text{cm}^2$ C $8\,\text{cm}^2$ D $10\,\text{cm}^2$ E $12\,\text{cm}^2$

3. What is the value of $1-(2-(3-4-(5-6)))$?

A -2 B -1 C 0 D 1 E 2

4. The diagram shows a square, its two diagonals and two line segments, each of which connects two midpoints of sides of the square.



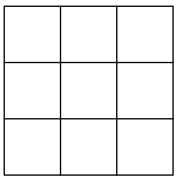
What fraction of the area of the square is shaded?

A $\frac{1}{8}$ B $\frac{1}{10}$ C $\frac{1}{12}$ D $\frac{1}{16}$ E $\frac{1}{24}$

5. We know that $1+2+3+4=10$. It is also true that $1^3+2^3+3^3+4^3=10^n$ for some integer n. What is this integer?

A 1 B 2 C 3 D 4 E 5

6. To draw a 3 by 3 square grid you need 8 straight lines, as shown.



How many straight lines do you need to draw a n by n square grid?

A $n+5$ B $3n-1$ C n^2-1 D $4(n-1)$ E $2(n+1)$

7. What is 0.015% of 60 million?

A 900 B 9000 C 90 000 D 900 000 E 9 000 000

8. $\sqrt{\sqrt{x}}=3$. What is the value of x?

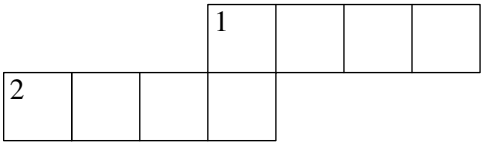
A $\sqrt{\sqrt{3}}$ B $\sqrt{3}$ C 9 D 12 E 81

9. Merryn wrote down the numbers 2, 0, 2, 3 and one further number. What was the median of her five numbers?

A 0 B 2 C 2.5 D 3
E more information required

10.

Across	**Down**
1. A power of 5	1. A power of 6
2. A power of 4	



Eight of the digits from 0 to 9 inclusive are used to fill the cells of the crossnumber. What is the sum of the two digits which are not used?

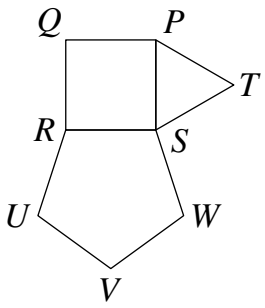
A 12 B 13 C 14 D 15 E 16

11. Jill was given a large jar of jam. She gave one sixth of the jam to Jan. Jill then gave one thirteenth of the remaining jam to Jas. Jill was left with 1 kg of jam.
What was the weight, in kg, of the jam in Jill's jar at the start?

A 1.2 B 1.3 C 1.4 D 1.6 E 1.9

12. In the diagram, $PQRS$ is a square, PST is an equilateral triangle and $SRUVW$ is a regular pentagon.

What is the size of angle WTS?



A 35° B 36° C 37° D 38° E 39°

13. The mean of p and q is 13; the mean of q and r is 16; the mean of r and p is 7.
What is the mean of p, q and r?

A 12 B 13 C 14 D 15 E 16

14. A regular octagon $PQRSTUVW$ has sides of length 2 cm. When I shade the rectangles $PQTU$ and $RSVW$, four small triangles inside the octagon remain unshaded. What is the total area, in cm^2, of these four triangles?

A 1 B 2 C 4 D 6 E 8

15. How many of the following polygons could exist?

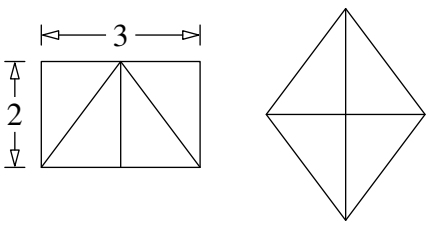
A triangle with all three sides the same length, but three different interior angles.
A quadrilateral with all four sides the same length, but four different interior angles.
A pentagon with all five sides the same length, but five different interior angles.

A only the pentagon
B only the quadrilateral
C the quadrilateral and the pentagon
D all three
E none of them

16. The sum of the lengths of the three sides of a right-angled triangle is 16 cm. The sum of the squares of the lengths of the three sides of the triangle is 98 cm^2.
What is the area, in cm^2, of the triangle?

A 8 B 10 C 12 D 14 E 16

17. A 3 by 2 rectangle is split into four congruent right-angled triangles, as shown in the left-hand diagram.
Those four triangles are rearranged to form a rhombus, as shown in the right-hand diagram.
What is the ratio of the perimeter of the rectangle to the perimeter of the rhombus?



A 3 : 2 B 2 : 1 C 1 : 1 D 1 : 2 E 2 : 3

18. How many squares are exactly four greater than a prime?

A 0 B 1 C 2 D 3 E 4

19. What is the positive difference between the numerator and the denominator when the expression shown is written as a single fraction in its simplest form?

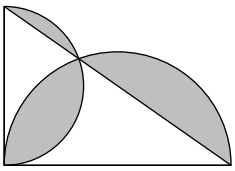
$$\frac{n}{n+1-\dfrac{n+2}{n+3}}$$

A $2n+2$ B $n+2$ C n D 2 E 1

20. I roll two standard six-sided fair dice. At least one of the scores obtained on the dice is 3. What is the probability that both of the scores on the dice are 3?

A $\frac{1}{12}$ B $\frac{1}{11}$ C $\frac{1}{6}$ D $\frac{1}{3}$ E $\frac{1}{4}$

21. A semicircle of radius 3 units is drawn on one edge of a right-angled triangle, and a semicircle of radius 4 units is drawn on another edge. The semicircles intersect on the hypotenuse of the triangle, as shown.
What is the shaded area, in square units?



A $\frac{25\pi}{2}-24$ B 12 C $\frac{25\pi}{2}-6$ D $25\pi-24$ E 24

22. The numbers x and y satisfy both of the equations

$$23x+977y=2023 \quad \text{and} \quad 977x+23y=2977.$$

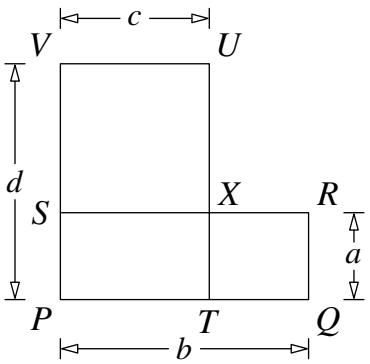
What is the value of x^2-y^2?

A 1 B 2 C 3 D 4 E 5

23. It is possible to choose, in two different ways, six different integers from 1 to 9 inclusive such that their product is a square. Let the two squares so obtained be p^2 and q^2, where p and q are both positive.
What is the value of $p+q$?

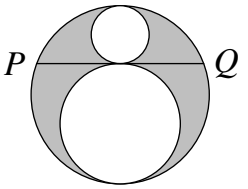
A 72 B 84 C 96 D 108 E 120

24. A rectangle $PQRS$ has side-lengths a and b, with $a<b$. The rectangle $PTUV$ has side-lengths c and d, with $c<d$. Also, $a<d$ and $c<b$, as shown. The sides RS and TU cross at X.
Which of these conditions guarantees that Q, X and V lie on a straight line?



A $\frac{a}{b}+\frac{c}{d}=1$ B $\frac{a}{c}+\frac{b}{d}=1$ C $\frac{a}{d}+\frac{c}{b}=1$
D $\frac{a}{c}+\frac{d}{b}=1$ E $\frac{c}{a}+\frac{b}{d}=1$

25. The diagram shows two unshaded circles which touch each other and also touch a larger circle. Chord PQ of the larger circle is a tangent to both unshaded circles. The length of PQ is 6 units.
What is the area, in square units, of the shaded region?



A 3π B $\frac{7\pi}{2}$ C 4π D $\frac{9\pi}{2}$ E 5π

