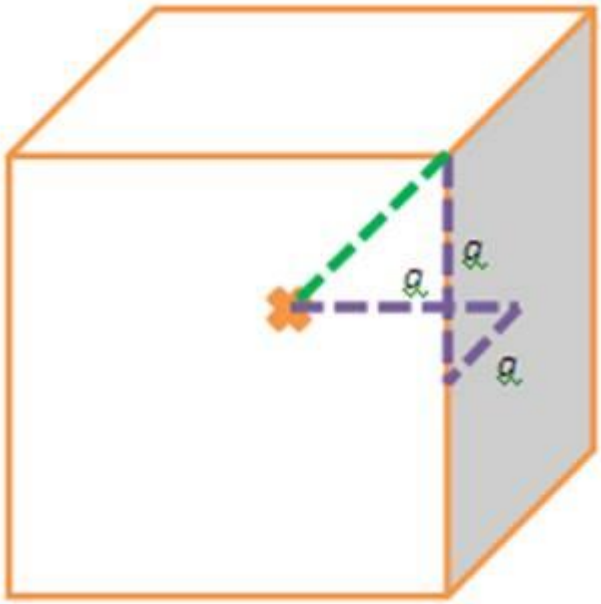
Challenge 10: Spheres and Cubes

Solution

Solved by five different solvers, including two puzzlers new to our community – well done to EJ Choi and Emilio Bello!

Here's an outline to the argument which everyone used. The key thing is visualizing the problem and then applying pythagoras' theorem.
Let the side length of the smaller cube be 2a, so that a is the length of half of one side of the cube.



In the diagram below the X marks the centre of the cube. We'd like to know the length of the green line, as this is a radius of the sphere in which the cube is sitting.

Each of the purple lengths is *a*, and using Pythagoras (twice!) we find that the green length is

$$\sqrt{3}a$$

Now notice that the green length, which is a radius of the sphere, must be the length of half of one side of the larger cube.

So the *length scale factor* from the smaller cube to the larger cube is

$$\sqrt{3}$$

Therefore the *volume scale factor* from the smaller cube to the larger cube is

$$\sqrt{3}^3 = 3\sqrt{3}$$

So the larger cube's volume is

$$3\sqrt{3}$$

times as large as the smaller cube's volume.