**Challenge 12: Grid Intersections**

**Solution**

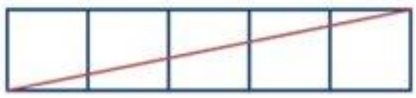
The challenge consisted of two questions: first, an introductory one:
"How many 1 x 1 squares does the diagonal line pass through in the 3 x 5 rectangle shown below?"

To which the answer, obviously, is 7. And a more general, proper one:
"How many 1 x 1 squares does the diagonal line of an n x m rectangle pass through? Make sure you justify your answer clearly."

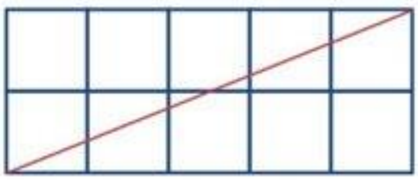
Important: For the purposes of this justification, I am presuming that a line passing through a vertex of a square without passing through the edge does not count as passing through a square.

Before I explain my reasoning, the answer to the problem is:
**(((n+m)/h)-1)*h**
where ***n*** and ***m*** are the side lengths of the rectangle or square, and ***h*** is the highest common factor of ***n*** and ***m***.

For the purposes of this justification, ***n*** should be smaller than or equal to ***m***.
Basically, if a rectangle is ***n*m*** when **n is 1**, it is obvious that the line will pass through every square:



= 5 travelled squares.



= 6 travelled squares.

Thus, the squares covered by the line would seem to be given by the formula ***n+m-1*** (the **-1** is present because if ***n=1***, no ascension squares are needed).
But this does not account for the cases where h is not equal to 1: the reason why we need to include h is because if ***HCF(n,m)*** is not equal to **1**, the line will pass through at least **1** internal vertex of the square. Thus, we have to split the square into smaller ones, evaluate one normally, and scale up to the full square.

Thus, the equation is ***(((n+m)/h)-1)*h***.