Challenge 13: Wordy Sums

Solution

First solution:

[Editor's note: an eagle eyed reader has noted that this solution isn't quite right, as the number represented by I and the number represented by R are both equal to 5, which breaks the rules! Bonus points to anyone who can spot precisely where student's reasoning breaks down...]

After some trial and error I realised that each column will have 1 integer left over representing a tenth. The maximum integer I could form by doubling the first single integers (1-9) is 18 meaning that it would be easier to include numbers with only tenths of 1 left over. This meant that T had to be 1.

1+1+1=3
2+2+1=5
3+3+1=7
4+4+1=9
5+5+1=11
6+6+1=13
7+7+1=15
8+8+1=17
9+9+1=19

The column with 'O' is the first column that consists of the same letter. So I needed to find a number that could be multiplied by two and added to 1 to form a two digit number ending with the number itself. Using my list above, I found that **9+9+1=19** meaning that O had to equal 9.
So far:

	D	O	U	B	L	E
		9				
	D	O	U	B	L	E
		9				
			T	O	I	L
			1	9		
T	R	O	U	B	L	E
1		9				

Next for the U,U,T,U column, I had find a number that I could multiply by 2 and add to 2 to equal a 2 digit number ending with the number itself. (the other '1' being from the 1 representing the tenth from the previous column). **8+8+1+1=18**

	D	O	U	B	L	E
		9	8			
	D	O	U	B	L	E
		9	8			
			T	O	I	L
			1	9		
T	R	O	U	B	L	E
1		9	8			

To find B, I had to find an integer that could be added to 9 and 1 to form a two digit number ending in the number itself. So I added the 9 and 1 from the previous column to get 10. The last number in 10 is 0 so by using 0 I get **0+0+9+1** which equals 10; a two digit number where the number can be added to 9 and 1 and last digit of the outcome is the same as the number itself.

	D	O	U	B	L	E
		9	8	0		
	D	O	U	B	L	E
		9	8	0		
			T	O	I	L
			1	9		
T	R	O	U	B	L	E
1		9	8	0		

Using my list of integers multiplied by 2 and added to one, and some trial and error, I found that **4+4+1=9** and **9+5=14**. Therefore:

	D	O	U	B	L	E
		9	8	0	4	
	D	O	U	B	L	E
		9	8	0	4	
			T	O	I	L
			1	9	5	4
T	R	O	U	B	L	E
1		9	8	0	4	

As E is the last column, there is no 1 representing a tenth left over. So I looked at my list and instead of adding 1, I added 4 (from L). I then found that **6+6+4=16**.

	D	O	U	B	L	E
		9	8	0	4	[illegible]
	D	O	U	B	L	E
		9	8	0	4	[illegible]
			T	O	I	L
			1	9	5	4
T	R	O	U	B	L	E
1		9	8	0	4	[illegible]

As R and D are letters that have not been used at all yet, I could pick any single digit number at random (that I haven't used yet) that could be multiplied by 2 to result in a 2 digit number between 10 and 19 (due to T already being 1 meaning that it had to be a number in the teen). 7 is the lowest number that I have not yet used that when doubled, equals a two digit number that includes integers that I have not yet used.

	D	O	U	B	L	E
	7	9	8	0	4	6
	D	O	U	B	L	E
	7	9	8	0	4	6
			T	O	I	L
			1	9	5	4
T	R	O	U	B	L	E
1	5	9	8	0	4	6

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  798046
 +798046
 +  1954
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 1598046
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Second solution

The 'units' column of the addition sum says that **E * 2 + L** is equal to some number with E as its last digit. This value could potentially be as large as 26 **(= 9 * 2 + 8)**, but this does not have a last digit of 9; in fact, all of the values of **E * 2 + L** above 19 do not have the last digit E, so they cannot work. Nor do any of the values below 10: these would, quite clearly, be smaller than **E * 2**, at least when L was factored in as well. Therefore, **9 < E * 2 + L < 20**, and the last digit of **E * 2 + L** is E, so **E * 2 + L** is equal to **E + 10**. This means that the 'units' column can be turned into this equation:

$$2E + L = E + 10$$

$$\therefore \quad L = 10 - E \quad \{\mathbf{1}\},$$

where I have called the resultant equation {1}. When this logic is applied to the next columns, a 1 must be added to the left side, because of the 'carried' value from the previous column. I have done this for the next three columns:

$$2L + I + 1 = L + 10$$

$$\therefore \quad I = 9 - L \quad \{\mathbf{2}\}$$

$$2B + O + 1 = B + 10$$

$$\therefore \quad O = 9 - B \quad \{\mathbf{3}\}$$

$$2U + T + 1 = U + 10$$

$$\therefore \quad T = 9 - U \quad \{\mathbf{4}\}$$

and have again called the resulting equations by the corresponding number, as they will turn out to be useful later. Now for the fifth column from the left:

$$2O + 1 = O + 10$$

$$\therefore \quad O = 9$$

By **{3}**, B = 0.

Now, O and the carried 1 are the only players: it follows directly that O must equal 9. I have also used {3} to show that B must equal 0.

Now let's have a look at the sixth column from the left. For the first time, the last digit of the sum is not used in the sum itself, so it can no longer be said that the result is more than nine:

2D+1=R (+10?)

However, if the column did add to less than 10, then there would be no carried digit, and T would have to equal 0, which would make T equal to B. Since this cannot be the case, the column must sum to more than 9:

$$2D + 1 = R + 10$$

$$\therefore \quad 2D = R + 9 \quad \{5\}$$

So, T = 1, and by {4}, U = 8.

Let's display what we've found so far in a table:

E	L	I	B	O	U	T	D	R
			0	9	8	1		

Clearly, E, L and I are going to be found together, and so are D and R. For E, L and I, we have equations {1} and {2}.

$$I = 9 - L \quad \{2\}$$

$$I = 9 - (10 - E)$$

$$I = E - 1 \quad \{6\}$$

Given E, we can now work out both L and I, using {1} and {3}. Here are the possible values, from 2 up to 7 as the others are already used up by B, O, U, and T:

E	L	I
~~2~~	~~8~~	~~1~~
3	7	2
4	6	3
~~5~~	~~5~~	~~4~~
6	4	5
7	3	6

The rows that are crossed out have either a number already used by B, O, U, or T, or have the same number twice, and are therefore illegal. What matter here are the left over values that could be taken by R or D. Here they are with the corresponding values of E:

E	Possible values of R and D
3	4, 5, 6
4	2, 5, 7
6	2, 3, 7
7	2, 4, 5

Now, let's look at {5} again. Clearly, R must be odd in order to make R + 9 even, which it must be, so R = 3, 5 or 7. For R = 3, {5} would give D = 6; for R = 5, {5} would give D = 7; and for R = 7, {5} would give D = 8, which is illegal, so R and D are either 3 and 6, or 5 and 7, respectively. Only one of the four rows in the table above has both 5 and 7, and none of them have both 3 and 6, so the E = 4 is the only possible row. So we now know that R = 5, D = 7, and E = 4. Finally, we can find the values of L and I in the E = 4 row of the E, L and I table: L = 6, and I = 3. So that completes the puzzle, and the values table:

E	L	I	B	O	U	T	D	R
4	6	3	0	9	8	1	7	5

Just to check: 798064 + 798064 + 1936 = 1598064. It works!