Challenge 77: Powerful Portents

Prove the following results by **hand**: in other words, do not use a calculator! This problem is all about careful reasoning using powers. You should think carefully about how to minimise the amount of arithmetic you have to do...

Here. we use the notation x^y to mean x^y.

1. Show that $3^{12} > 2^{19}$.

2. Hence show that $(1.5)^q > 2$, where $q = \sqrt{3}$.

3. Hence carefully show that if $x^y = 2$, where $y = x^x$, then $x < 1.5$. It's important to be really clear in your argument here!

And, as an extra titbit: the first of these statements is related to the fact that in musical theory, 12 perfect fifths is slightly more than seven octaves. If you know anything about musical theory, you might want to think about why...