Mathematical Logic & Dirichlet's Principle

Session 1

Tutor: Jude Mbroh

February 22nd, 2025

Albion Online Intellectual Club

Table of contents

Introduction

Tutor Introduction

Tutor: Jude Mbroh
University: Imperial College London
Degree: MEng Aeronautical Engineering
Experience: Flight Dynamics Technical Lead at ICL Rocketry; tutored students in A-Level Maths & Further Maths

Content for Mathematical Logic

The series of sessions will cover multiple areas of maths within the logic problems so students can form connections between topics.

- Core Focus: Teach students the basics of logical reasoning, truth tables, logical operators (AND, OR, NOT, IMPLIES), and problem-solving strategies.
- Number Theory: Use logic puzzles involving divisibility, prime numbers, or modular arithmetic.
- Geometry: Introduce logic problems related to geometric shapes or properties.
- Algebra: Create logic puzzles based on solving equations or inequalities.
- Probability: Incorporate logic problems involving probability (applications in medicine or other real-world themes).

Content for Dirichlet's Principle

Questions will be introduced using multiple different branches of mathematics.

- Combinatorics: Use the principle to solve counting problems.
- Number Theory: Apply the principle to problems involving remainders or divisibility.
- Geometry: Use the principle to solve problems about points, lines, or shapes.
- Applications: Use a chessboard to demonstrate the principle.

Mathematical Logic

Introduction to Mathematical Logic

What is Mathematical Logic?

- A systematic way to analyse and solve problems using reasoning.
- Helps us understand how to make valid arguments and draw conclusions.

Example: Logical Operators

P	Q	$P \wedge Q$ (AND)
T	T	T
T	F	F
F	T	F
F	F	F

Task 1 - The Monty Hall Problem

Problem Statement:

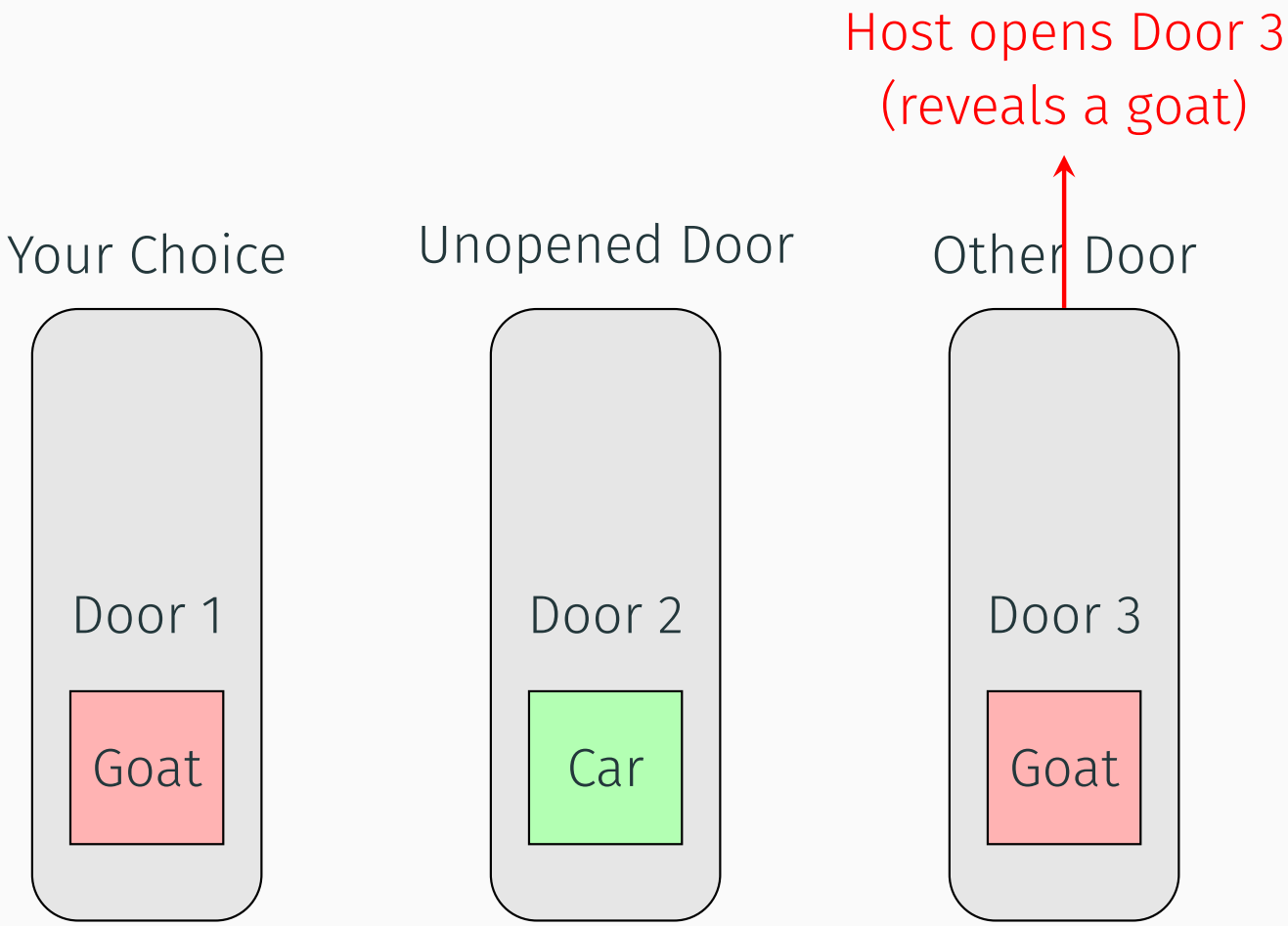
You are a contestant on a game show. You are presented with three doors:

- Behind one door is a **car** (the prize you want).
- Behind the other two doors are **goats**.

The Game Proceeds as Follows:

1. You pick one of the three doors (e.g., Door 1).
2. The host, who knows what's behind each door, opens one of the remaining two doors, revealing a goat (e.g., Door 3).
3. The host then asks you: "Do you want to **stay with your original choice** or **switch to the other unopened door**?"

Task 1 - Monty Hall Problem: Visualisation



Should you switch or stay?

Task 2 - Military Band Formation Puzzle

Problem Statement:

A very large military band marched in a **square formation** on a parade ground. They then regrouped into a **rectangle** such that the number of rows increased by 5.

Question: How many musicians were in the band?

Visualisation:

- **Square Formation**: $n \times n$
- **Rectangle Formation**: $(n + 5) \times m$

Task 3 - Deductive Reasoning

Problem Statement:

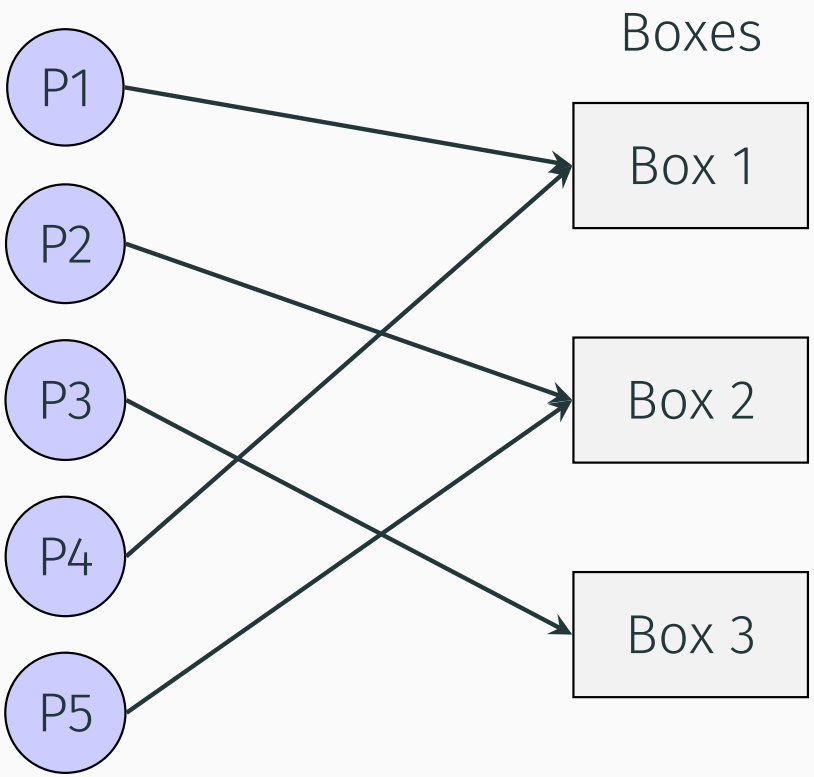
- Both Alice and Bob have a positive integer drawn on their foreheads.
- Each can see the other's number but not their own.
- An observer tells them: *"The number 50 is either the sum or the product of the two numbers."*
- Alice says: *"I do not know my number."*
- Bob replies: *"I do not know my number either."*

Question: What is Alice's number?

Dirichlet's Principle

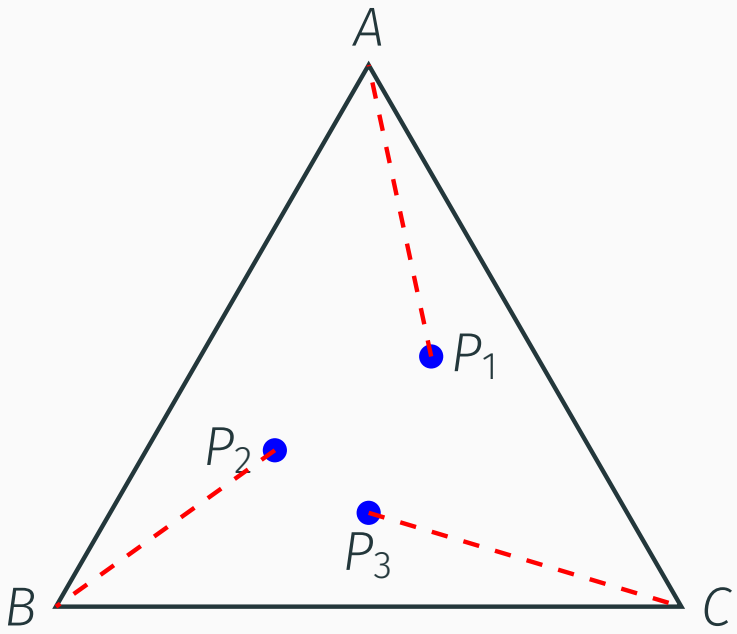
Introduction to Dirichlet's Principle

Definition: In combinatorics, Dirichlet's box principle states that if n balls are to be placed into k boxes, and $n > k$, then at least one box must contain more than one ball.



Task 4 - Geometry

Prove that in any group of 5 points placed inside an equilateral triangle of side length 1, at least two points must be at most $\frac{1}{2}$ units apart.



Task 5 - Sequences

In the English language, what is the smallest number of words in a sequence such that at least two words must start with the same letter?

Question: How do you determine this number for every language?

Task 6 - Aliens Visiting Earth

Some aliens are visiting Earth. The captain of their ship wants to send explorers to all 195 countries. Given that each explorer goes to a country at random, how many explorers must be sent out to guarantee that there will be at least two explorers in the same country?

Artwork by Nathan W. Pyle