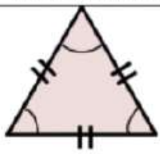
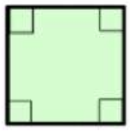
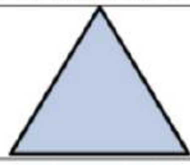
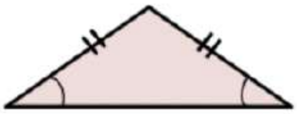
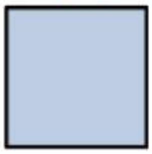
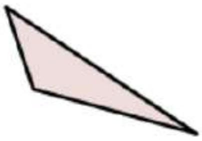
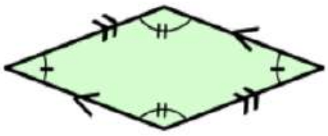
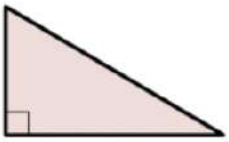
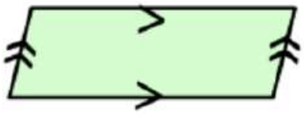
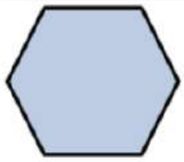
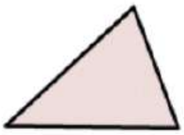
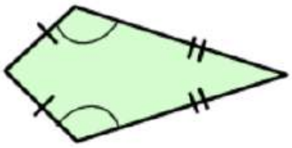
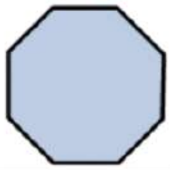
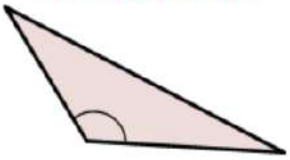
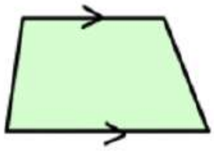
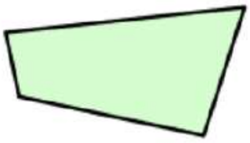
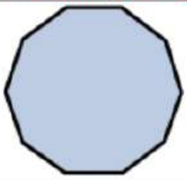
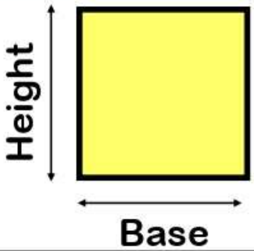
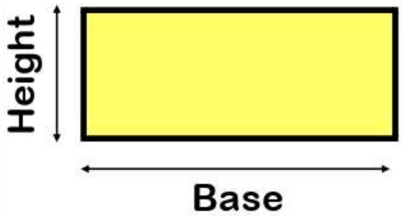
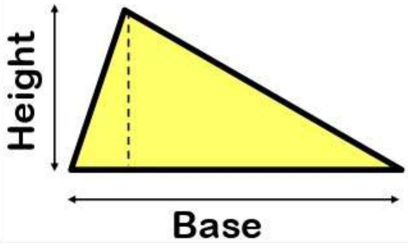
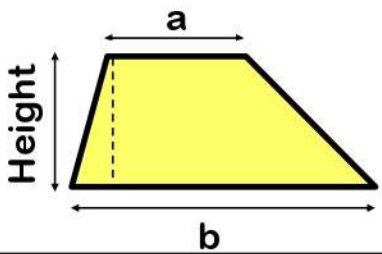
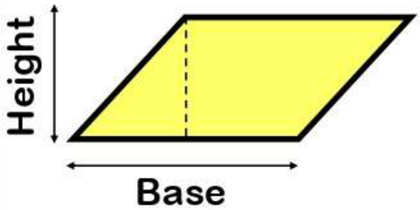
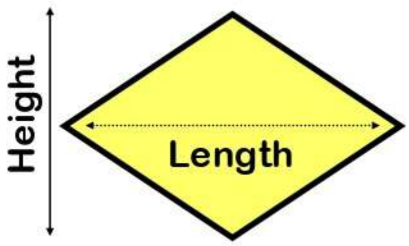
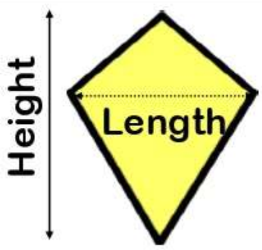
GEOMETRY QUICK GUIDE 2: 2D SHAPES (UK)

TRIANGLES	QUADRILATERALS	REGULAR POLYGONS
		
Equilateral triangle All sides equal; interior angles 60°	**Square** All sides equal; all angles 90°	**Equilateral triangle** 3 sides; angle 60°
		
Isosceles triangle 2 sides equal; 2 congruent angles	**Rectangle** Opposite sides equal, all angles 90°	**Square** 4 sides; angle 90°
		
Scalene triangle No sides or angles equal	**Rhombus** All sides equal; 2 pairs of parallel lines; opposite angles equal	**Regular Pentagon** 5 sides; angle 108°
		
Right triangle 1 right angle	**Parallelogram** Opposite sides equal, 2 pairs of parallel lines	**Regular Hexagon** 6 sides; angle 120°
		
Acute triangle All angles acute	**Kite** Adjacent sides equal; 2 congruent angles	**Regular Octagon** 8 sides; angle 135°
	 	
Obtuse triangle 1 obtuse angle	**Trapezium** 1 pair of parallel sides **Trapezoid** No pairs of parallel sides	**Regular Decagon** 10 sides; angle 144°

Formula Sheet for Area of 2D Shapes

Shape	Name	Formula for Area
	Square	Base x Height
	Rectangle	Base x Height
	Triangle	Base x Perpendicular Height ÷ 2
	Trapezoid	$\frac{(a + b) \times \text{height}}{2}$
	Parallelogram	Base x Perpendicular Height
	Rhombus	Length x Height ÷ 2
	Kite	Length x Height ÷ 2

48 PROBABILITY

What is probability?

Probability is the branch of mathematics that allows you to work out how likely or unlikely an **outcome** or result of an **event** might be.

In probability, an outcome that is certain to happen has a probability of 1 and an event that is impossible has a probability of 0. Probabilities greater than 1 or less than 0 have no meaning.

Theoretical probability is based on equally likely outcomes. You can use it to tell how an event should perform in theory, whereas **experimental probability** (or **relative frequency**) tells you how an event performs in an experiment.

Events and outcomes

An **event** is something that happens, such as throwing a die, or tossing a coin, or picking a card from a pack.

An **outcome** is the result of an event, such as scoring a 3 or a 6 when you throw a die.

If the outcome is the required result, such as throwing a 6 to start a game, then the outcome is a **success**.

The **probability** is a measure of how likely an outcome is to happen.

In general:

$$\text{probability of success} = \frac{\text{number of 'successful' outcomes}}{\text{number of 'possible' outcomes}}$$

and you can use p(success) as shorthand for the probability of success.

$$\text{p(success)} = \frac{\text{number of 'successful' outcomes}}{\text{number of 'possible' outcomes}}$$

Using a die

Worked example

A box contains 25 coloured balls, where seven balls are red, ten balls are blue and eight balls are yellow. A ball is selected from the box at random. Calculate the probability of selecting:

a) a red ball **b)** a blue ball **c)** a red or a yellow ball

d) a red or a blue or a yellow ball **e)** a green ball.

Use:

$$\text{p(success)} = \frac{\text{number of 'successful' outcomes}}{\text{number of 'possible' outcomes}}$$

a) $\text{p(red ball)} = \frac{\text{number of red balls}}{\text{number of balls}} = \frac{7}{25}$

b) $\text{p(blue ball)} = \frac{\text{number of blue balls}}{\text{number of balls}} = \frac{10}{25} = \frac{2}{5}$ Cancelling to lowest terms.

HINT

- You could also give the answer as a decimal or a percentage.
  So p(red ball) = 0.28
  or p(red ball) = 28%

c) p(red or yellow ball) $= \frac{\text{number of red or yellow balls}}{\text{number of balls}}$

$= \frac{15}{25} = \frac{3}{5}$ Cancelling to lowest terms.

d) p(red or yellow or blue ball) $= \frac{\text{number of red or yellow or blue balls}}{\text{number of balls}}$

$= \frac{25}{25}$

$= 1$ Meaning that this outcome is certain to happen.

e) p(green ball) $= \frac{\text{number of green balls}}{\text{number of balls}} = \frac{0}{25}$

$= 0$ Meaning that this outcome is impossible, it cannot happen.

TOTAL PROBABILITY

The probability of an outcome happening is equal to 1 minus the probability of the outcome not happening.

p(outcome occurs) = 1 − p(outcome does not occur)

HINT

- You may see the word 'dice' used instead of 'die' in these questions.
  Die is the singular form, dice is the plural.

Worked example

The probability that it will rain tomorrow is $\frac{1}{5}$. What is the probability that it will not rain tomorrow?

p(rain) $= \frac{1}{5}$

p(not rain) $= 1 - \frac{1}{5} = \frac{4}{5}$ p(not rain) = 1 − p(rain)

Possibility spaces

A **possibility space** is a diagram which can be used to show the outcomes of various events.

Worked example

Two fair dice are thrown and the sum of the scores on the faces is noted. What is the probability that the sum is 8?

Draw a diagram to illustrate the possible outcomes.

The diagram to illustrate the possible outcomes is shown below. There are 36 possible outcomes. There are 5 outcomes that give a total of 8.

Probability that the sum is 8 = $\frac{5}{36}$.

		Second die					
		1	2	3	4	5	6
First die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Theoretical and experimental probability

Worked example

A die is thrown 100 times.

a) How many times would you expect to throw a six?

This frequency distribution is obtained.

Score	1	2	3	4	5	6
Frequency	18	15	19	17	16	15

b) What is the relative frequency of a score of 6?

c) What is the relative frequency of getting an even number?

d) For which score are the theoretical probability and relative frequency the closest?

a) When throwing a die 100 times:
expected number of sixes $= 100 \times \frac{1}{6} = 16.666\,66\ldots$

$= 17$ (to the nearest whole number)

b) The relative frequency of a score of 6 is $\frac{15}{100} = \frac{3}{20}$.

c) The frequency of getting an even number is $15 + 17 + 15 = 47$.

The relative frequency of getting an even number is $\frac{47}{100}$.

d) The theoretical and experimental probabilities are closest for a score of 4.

HINT

- The term relative frequency is used to describe the experimental probability.

QUESTIONS

Q1 A box contains 50 balls coloured blue, red and green. The probability of getting a blue ball is 32% and the probability of getting a red ball is 0.46.

a) How many blue balls are there in the box?

b) How many red balls are there in the box?

c) How many green balls are there in the box?

Q2 The probability that a train arrives early is 0.2 and the probability that the train arrives on time is 0.45. What is the probability that the train arrives late?

Q3 An experiment consists of throwing a die and tossing a coin. Draw a possibility space for the two events and use this to calculate the probability of scoring:

a) a head and a 1

b) a tail and an odd number.

Q4 Two tetrahedral dice, each numbered 1 to 4, are thrown simultaneously. Draw a possibility space for the total of the two dice and use this information to calculate the probability of scoring a total of:

a) 2 **b)** 6 **c)** 9.

What is the most likely outcome?

Q5 A die is thrown 120 times. What is the expected frequency of a number greater than 4?

More questions on the CD ROM

Q6 The probability that a new car will develop a fault in the first month after delivery is 0.062%. A garage sells 1037 new cars in one year. How many of these cars will be expected to develop a fault in the first month after delivery?

Answers are on page 227.

49 THE ADDITION AND MULTIPLICATION RULES

Two or more events are **mutually exclusive** if they cannot happen at the same time. Two events are **independent** if the outcome of one does not affect the outcome of the other.

Mutually exclusive events

THE ADDITION RULE

When you are working with **mutually exclusive** events you can apply the **addition rule** (also called the **or rule**) which states that:

$p(A \text{ or } B) = p(A) + p(B)$

and for more than two mutually exclusive events:

$p(A \text{ or } B \text{ or } C \text{ or } \ldots) = p(A) + p(B) + p(C) + \ldots$.

Worked example

A spinner with ten sides, numbered 1 to 10, is spun. What is the probability of getting:

a) a five

b) a five or a six

c) a multiple of 3 or a multiple of 4

d) a multiple of 2 or a multiple of 3?

a) $p(5) = \frac{1}{10}$

b) $p(5 \text{ or } 6) = p(5) + p(6)$ As the events are mutually exclusive.

$= \frac{1}{10} + \frac{1}{10}$

$= \frac{2}{10}$

$= \frac{1}{5}$ Cancelling down to the lowest terms.

c) p(multiple of 3 or multiple of 4)

$= p(3) + p(6) + p(9) + p(4) + p(8)$ As the events are mutually exclusive.

$= \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{5}{10}$

$= \frac{1}{2}$ Cancelling down to the lowest terms.

d) p(multiple of 2 or multiple of 3)

= p(2 or 3 or 4 or 6 or 8 or 9 or 10)

As the events are not mutually exclusive, because the number 6 is common to both events and if the probabilities are added then this probability will be added twice.

$= p(2) + p(3) + p(4) + p(6) + p(8) + p(9) + p(10)$

As these events are now mutually exclusive.

$= \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$

$= \frac{7}{10}$

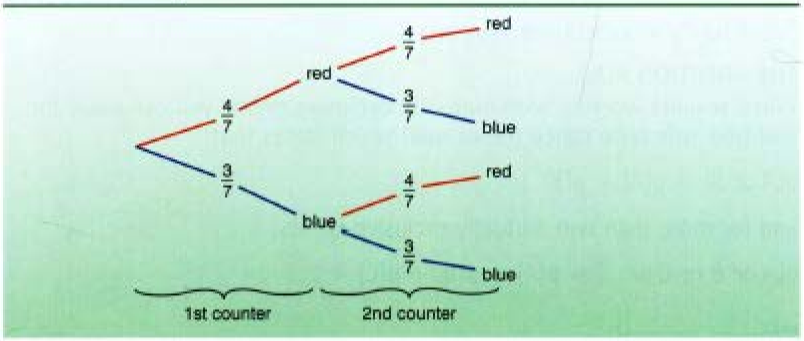
Independent events

TREE DIAGRAMS

In a **tree diagram**, you write the probabilities of the outcomes of different events on different branches of the 'tree'.

Worked example

A bag contains four red and three blue counters. A counter is drawn from the bag, replaced and then a second counter is drawn from the bag. Draw a tree diagram to show the various possibilities that can occur.



THE MULTIPLICATION RULE

For **independent events** you can use the **multiplication rule** (also called the **and rule**) which states that:

$p(A \text{ and } B) = p(A) \times p(B)$

Similarly for more than two independent events:

$p(A \text{ and } B \text{ and } C \text{ and } \ldots) = p(A) \times p(B) \times p(C) \times \ldots$

Worked example

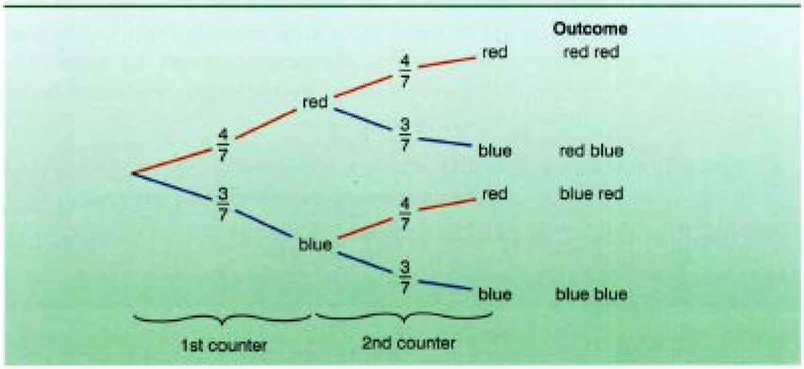
A bag contains four red and three blue counters. A counter is drawn from the bag, replaced and then a second counter is drawn from the bag. Draw a tree diagram and use it to calculate the probability that:

a) both counters will be red

b) both counters will be blue

c) the first counter will be red and the second counter blue

d) one counter will be red and one counter will be blue.



a) p(red and red) = p(red) × p(red) As the events are independent.

$= \frac{4}{7} \times \frac{4}{7} = \frac{16}{49}$

b) p(blue and blue) = p(blue) × p(blue) As the events are independent.

$= \frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$

c) p(red and blue) = p(red) × p(blue) As the events are independent.

$= \frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$

d) p(one counter will be red and one counter will be blue) is the same as p(red and blue or blue and red).

= p (red and blue) + p(blue and red). As the events are mutually exclusive.

= p(red) × p(blue) + p(blue) × p(red) As the events are independent.

$= \frac{4}{7} \times \frac{3}{7} + \frac{3}{7} \times \frac{4}{7} = \frac{12}{49} + \frac{12}{49} = \frac{24}{49}$

Both of these outcomes give one red and one blue.

QUESTIONS

Q1 A die with faces numbered 1 to 6 is rolled and the value on the face uppermost is noted. Find the probability that the result will be:

a) a 5 or a 6 **b)** an even number

c) a factor of 8.

Q2 Letters are chosen from the word:

PROBABILITY

Find the probability that the chosen letter is:

a) the letter P **b)** the letter B

c) the letter B or the letter I.

Q3 A counter is selected from a box containing three red, four green and five blue counters and a second counter is selected from a different box containing five red and four green counters. Draw a tree diagram to show the various possibilities when a counter is drawn from each bag.

Q4 The probability that a car will fail its safety test because of the lights is 0.32 and the probability that a car will fail the safety test because of the brakes is 0.55. Calculate the probability that the car fails because of:

a) its lights and its brakes

b) its lights only.

Q5 The probability that a particular component will fail is 0.015. Draw and label a tree diagram to show the possible outcomes when two such components are chosen at random. Calculate the probability that:

a) both components will fail

b) exactly one component will fail.

More questions on the CD ROM

Answers are on page 228.

50 Multiplication rule for dependent events

What are dependent events?

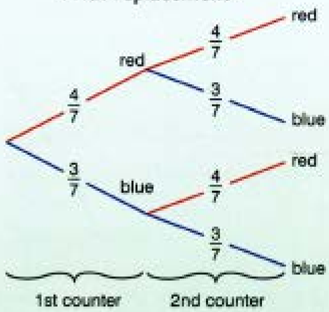
Two or more events are **dependent** if one event affects the probability of the other event.

The multiplication rule

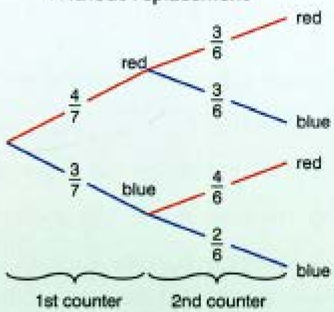
Worked example

A bag contains four red and three blue counters. A counter is drawn from the bag and then a second counter is drawn from the bag. Draw a tree diagram to show the various possibilities that can occur and use the diagram to find the probability that both counters are blue.

The question does not make it clear whether the first counter is replaced before the second counter is drawn. This gives rise to two possibilities as shown in the following tree diagrams.



Independent events



Dependent events

With replacement (independent events)

If the first counter is replaced before the second counter is drawn, then the two events are independent and the probabilities for each event are the same.

From the diagram, the probability that both counters are blue

= p(blue counter drawn first and blue counter drawn second)

= p(blue counter drawn first) × p(blue counter drawn second)

$= \frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$

Without replacement (dependent events)

If the first counter is not replaced before the second counter is drawn, then the two events are not independent (i.e. they are dependent) and the probabilities on the second event will be affected by the outcomes on the first event.

For the second counter:

if the first counter was blue, there are now two blue counters and six counters altogether

if the first counter was not blue, there are still three blue counters and six counters altogether.

From the diagram you can see that:

the probability that both counters are blue

= p(blue counter drawn first and blue counter drawn second)

= p(blue counter drawn first) × p(blue counter drawn second)

$= \frac{3}{7} \times \frac{2}{6} = \frac{6}{42} = \frac{1}{7}$

QUESTIONS

Q1 Mohammed has ten black and six brown socks in his drawer. If he removes two socks from the drawer, one after the other, calculate the probability that:

a) both socks are black

b) both socks are brown

c) the socks are different colours.

Q2 The probability that Sara passes the driving theory test on her first attempt is $\frac{6}{7}$. If she fails then the probability that she passes on any future attempt is $\frac{7}{8}$.

Draw a tree diagram to represent this situation and use it to calculate the probability that Sara passes the driving test on her third attempt.

Q3 A bag contains three black beads, five red beads and two green beads. Grace takes a bead at random from the bag, records its colour and replaces it. She does this two more times.

Work out the probability that, of the three beads Grace takes, exactly two are the same colour.

More questions on the CD ROM

Answers are on page 229.