Preface

Paradoxes come in all shapes and sizes. Some are straightforward paradoxes of logic with little potential for investigation, while others sit atop icebergs of entire scientific disciplines. Many can be resolved by careful consideration of their underlying assumptions, one or more of which may be faulty. These, strictly speaking, should not be referred to as paradoxes at all, since once a puzzle is solved it ceases to be a paradox.

A true paradox is a statement that leads to a circular and self-contradictory argument, or describes a situation that is logically impossible. But the word "paradox" does tend to be used more broadly to include what I prefer to call "perceived paradoxes." For such puzzles there is a way out. It may be that the paradox has hidden within it a trick or sleight of hand that deliberately misleads the listener or reader. Once the trick is uncovered, the contradiction or logical absurdity disappears. Another type of perceived paradox is one in which the statement and the conclusions, while initially sounding absurd or at the very least counterintuitive, turn out on more careful consideration not to be so, even if the result remains somewhat surprising.

And then there is the category of paradoxes in physics. All of these—well, nearly all —can be resolved with a little bit of fundamental scientific knowledge; and these are the ones that I focus on in this book.

So let us first take a brief look at a true logical paradox, just so that it is clear what I am *not* going to be talking about. This is a statement that is constructed in such a way that there really is no way out of the loop.

Take the simple assertion: "This statement is a lie." On first reading, I imagine the words themselves will seem straightforward enough. Think about their meaning, however, and the logical paradox will become evident as you work carefully through the statement's implications. Can five simple words really give you a headache? If so, I would argue that it is a fun sort of headache, which is perhaps itself a paradox, and one that you will no doubt feel sadistically obliged to pass on to family and friends.

You see, "This statement is a lie" is telling you that in announcing itself to be a lie it must itself be a lie, and so it is not a lie—in which case it is true, which is to say that it really is a lie, which means it is not a lie, and so on in an infinite loop.

There are many such paradoxes. This book is not about them.

This book is instead about my own personal favorite puzzles and conundrums in science, all of which have famously been referred to as paradoxes, but which turn out not to be paradoxes at all when considered carefully and viewed from the right angle.

While powerfully counterintuitive when first described, they always turn out to be missing some subtle consideration of the physics which, when taken into account, knocks out one of the pillars on which the paradox is built and brings the whole edifice toppling down. Despite having been resolved, many of them continue to be referred to as paradoxes, partly owing to the notoriety they gained when first articulated (before we had figured out where we were going wrong) and partly because, so presented, they are useful tools in helping scientists get their heads around some rather complex concepts. Oh, and because they are such delicious fun to explore.

Many of the puzzles we will look at do indeed seem at first to be true paradoxes rather than just perceived ones. That is the point. Take a simple version of the famous time travel paradox: what if you were to go back into the past using a time machine and kill your younger self? What happens to the killer you? Do you pop out of existence because you stopped yourself from growing older? If so, and you never did reach the age at which you became a murderous time traveler, then who killed the younger you? The older you has the perfect alibi: you never even existed! So if you did not survive to travel back in time and kill your younger self, then you do not kill your younger self and so you do survive to grow older and travel back in time and kill yourself, so you do that, so you don't survive, and so on. This appears to be the perfect logical paradox. And yet physicists have not yet ruled out the possibility, certainly in theory, of time travel. So how can we extricate ourselves from this paradoxical loop? I will discuss this particular paradox in .

Not all perceived paradoxes require a scientific background to make sense of them. To demonstrate this, I have given over the first chapter to a handful of such perceived paradoxes that can be resolved with commonsense logic. What do I mean by this? Well, consider a simple statistical paradox in which it is quite possible to draw the wrong conclusion from a basic correlation: it is known that towns with larger numbers of churches generally have higher crime rates. This is somewhat paradoxical, unless you believe that churches are breeding grounds for lawlessness and crime—which, whatever your religious and moral views, seems pretty unlikely. But the resolution is straightforward. Both a higher number of churches and a higher absolute level of crime are the natural results of a larger population. Just because A leads to B and A leads to C does not mean that B leads to C or vice versa.

Here is another example of a simple brainteaser that sounds paradoxical when first stated, but whose paradoxical nature dissolves away once it has been explained. It was recounted to me a few years ago by a Scottish professor of physics who is a colleague and close friend of mine. He claims that "every Scotsman who travels south to England raises the average IQ of both countries." The point is this: since all Scotsmen claim to be smarter than all Englishmen, then any one of them would enhance the average IQ of England by living there; however, to leave Scotland is such a foolish act that only those less intelligent among them would do so, leaving the average of those remaining

slightly higher. So you see, at first glance it sounds like a paradoxical statement, yet with simple logical reasoning it can be resolved beautifully—if not convincingly for the English, of course.

Once we have had some fun in Chapter 1 with a few well-known paradoxes that can be resolved without any science, we will move on to my nine chosen paradoxes in physics. After stating each one, I will lay it bare and explain how it evaporates away to reveal the underlying logic that shows its fallacy, or why it is not really an issue at all. They are all fun because they have some intellectual meat, and because *there is a way out*. You just need to know where to look, where to find the Achilles' heel that can be exploited with careful prodding and a better understanding of the science, until the paradox is a paradox no more.

The names of some of these paradoxes will be familiar. Take the Paradox of Schrödinger's Cat, for instance, in which an unfortunate feline is locked into a sealed box and is simultaneously both dead and alive until we open the box. Less familiar, perhaps, but still known to some, is Maxwell's demon, the mythical entity that presides over another sealed box and which is seemingly able to violate that most sacred of commandments in science, the Second Law of Thermodynamics—forcing the contents of the box to unmix and become ordered. To understand such paradoxes, and their resolution, it is necessary to grasp some background science; and so I have set myself the challenge of getting these scientific concepts across with as little fuss as possible, so that you can appreciate and enjoy the implications without any in-depth knowledge of calculus, thermodynamics, or quantum mechanics.

I have plucked several of the other paradoxes in this book from the undergraduate course on relativity that I have taught for the past fourteen years. Einstein's ideas on space and time, for instance, provide fertile ground for logical brainteasers such as the Pole in the Barn Paradox, the Paradox of the Twins, and the Grandfather Paradox. Others, such as those involving the cat and the demon, have, in the eyes of some, yet to be satisfactorily laid to rest.

When choosing my greatest enigmas in physics, I have not homed in on the biggest unsolved problems—for example, what dark matter and dark energy, which between them make up 95 percent of the contents of our universe, are made of, or what, if anything, was there before the Big Bang. These are incredibly difficult and profound questions to which science has yet to find answers. Some, like the nature of dark matter, that mysterious stuff that makes up most of the mass of galaxies, may well be answered in the near future if particle accelerators like the Large Hadron Collider in Geneva continue to make new and exciting discoveries; others, like an accurate description of a time before the Big Bang, may remain unanswered forever.

What I have aimed to do is make a sensible and broad selection. All the paradoxes I discuss in the following chapters deal with deep questions about the nature of time and space and the properties of the Universe on the very largest and smallest of scales.

Some are predictions of theories that sound very strange on first encounter, but which become intelligible once the ideas behind the theory are explored carefully. Let's see if we can't lay them all to rest, and along the way give you, dear reader, some mind-expanding fun.

1

THE GAME SHOW PARADOX

•———————•———•——•—•••

Simple probabilities that can really blow your mind

Before I get stuck into the physics, I thought I would lead you in gently with a few simple, entertaining, and frustrating puzzles as a warm-up. I should say that, in common with the rest of the collection in this book, none of these are real paradoxes at all; they just need to be considered carefully and unpicked. But unlike those coming later, which will require an understanding of the underlying physics, the paradoxes in this chapter are a collection of logical brainteasers that can be resolved without any scientific background at all. The last and most delicious of the group, known as the Monty Hall Paradox, is so utterly baffling that I will invest considerable effort in unpicking and analyzing it in several different ways so you can choose which particular solution you prefer.

All the puzzles in this chapter fall into one of two categories with the exotic-sounding names of "veridical" and "falsidical" paradoxes. A veridical paradox is one leading to a conclusion that is counterintuitive because it goes against common sense, and yet can be shown to be true using careful, often deceptively simple, logic. In fact, with these the fun lies in trying to find the most convincing way of demonstrating that it is true, despite that lingering uncomfortable feeling that there has to be a catch somewhere. The Birthday Paradox and the Monty Hall Paradox are both in this category.

A falsidical paradox, on the other hand, starts off perfectly sensibly, yet somehow ends up with an absurd result. However, in this case the apparently absurd result is indeed false, thanks to some subtly misleading or erroneous step in the proof.

Examples of falsidical paradoxes are the mathematical tricks that, by following a few steps of algebra, "prove" something like 2 = 1. No amount of logic or philosophizing should convince you that this can be true. I won't go into any of these here, mainly since I don't really want to be hitting you with algebra just in case you don't love it as much as I do. Suffice it to say that the calculation leading to the "solution" usually involves a step in which a quantity is divided by zero—something any self-respecting mathematician knows to avoid at all costs. Instead, I will focus on a few problems that you can appreciate with only the bare minimum of mathematical ability. I'll begin with

two great falsidical paradoxes: the Riddle of the Missing Dollar and Bertrand's Box Paradox.

THE RIDDLE OF THE MISSING DOLLAR

This is a brilliant puzzle that I used a few years ago when I was a guest on a TV quiz show called *Mind Games*—not that I am claiming that I was the first to come up with it, of course. The premise of the show was that each week the guests would compete against each other to solve puzzles set by the host of the show, the mathematician Marcus du Sautoy. In addition, each was expected to bring along their favorite brainteasers to try to bamboozle the other team.

Here is how it goes:

Three travelers check into a hotel for the night. The young man at the reception desk charges them $30 for a room with three beds in it. They agree to split the price of the room equally, each of them paying $10. They take the key and head up to the room to settle in. After a few minutes the receptionist realizes he has made a mistake. The hotel has a special offer on all week and he should only have charged them $25 for the room. So as not to get into trouble with his manager, he quickly takes five dollar bills from the till and rushes up to rectify his error. On the way to the room he realizes that he cannot split the five dollars equally between the three men, so he decides to give each of them one dollar and keep two for himself. That way, he argues, everyone is happy. Here, then, is the problem we are left with: each of the three friends will have contributed $9 toward the room. That makes $27 that the hotel has made, and the receptionist has a further $2, which makes $29. What has happened to the last dollar out of the original $30?

You may be able to see the solution to this straightaway; I certainly didn't when I first heard it. So I will let you think about it a little before you read on.

Have you worked it out? You see, this puzzle only sounds paradoxical because of the misleading way it is stated. The error in the reasoning is that I added the $27 dollars to the $2 taken by the receptionist—and there is no reason to do that, because there is no longer a total of $30 that needs to be accounted for. The receptionist's $2 should be *subtracted* from the $27 paid by the friends, leaving $25, which is the amount in the till.

BERTRAND'S BOX PARADOX

My second example of a falsidical paradox is credited to the nineteenth-century French mathematician Joseph Bertrand. (It is not his most famous paradox, which is rather more mathematically technical.)

You have three boxes, each containing two coins; each box is divided into two halves by a partition, with a coin in each half. Each side can be opened separately to see the coin inside (that is, without allowing you to see the other coin). One box contains two gold coins (we will call this one GG), the second (which we will call SS) contains two silver coins, and the third (GS) contains one of each. What is the probability of picking the box with the gold and silver coins in it? The answer is simple, of course: one in three. That is not the puzzle.

Figure 1.1 Bertrand's Boxes
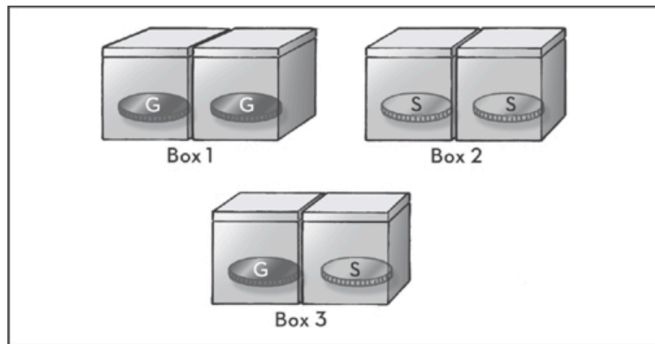
Now, pick a box at random. What if you open one of its lids and find a gold coin inside? What now are the chances of this box being the GS one? Well, since in finding a gold coin you know this cannot be the SS box, you must rule that option out and be left with two choices: either it is the GG box or it is the GS box. Hence the probability of its being the GS box is one in two, right?

Had you opened the lid to find a silver coin instead, then you could now rule out the GG option; so you are left with SS or GS, so the probability that this is the GS box is still one in two.

Since you must find either a gold or a silver coin when you open the lid of the chosen box, and since there are three coins of each kind in total, giving you an equal chance of finding either, there is therefore a one in two probability that you have found the GS box whatever coin you find. Thus, after taking a peek inside one half of your chosen box the overall probability that it is the GS box must change from one in three, as it was at the start, to one in two. But how can seeing one of the coins change the probability like this? If you choose a box at random and, before opening one of its lids, you know that there is a one in three chance that it is the GS one, then how, by seeing one of the coins inside, and gaining no information at all from this, *since you know you are certain to find either a gold or a silver coin anyway*, does the probability switch from one in three to one in two? Where are we going wrong?

The answer is that the probability is always one in three and never one in two, whether you see one of the coins in the box or not. Consider the case when you find a gold coin inside your box. There are three gold coins in total—let us call them Gl, G2 and G3, and let us say that the GG box contains coins Gl and G2, while G3 is the gold coin in the GS box. If you open one of the boxes and find a gold coin inside, there is a two in three chance of your having picked the GG box, since the coin you are looking at could be either G1 or G2. There is only a one in three chance that it is the G3 coin and therefore that the box you have picked is the GS one.

THE BIRTHDAY PARADOX

This is one of the best-known veridical paradoxes. Unlike the last two examples, there is no trick here, no error in the reasoning or sleight of hand in the telling. Whether you are convinced by the solution or not, I must stress that it is perfectly correct and consistent, both mathematically and logically. In a way, this frustration makes the paradox all the more fun.

Here is how it is stated:

How many people would you say there would have to be in a room for the chances of any two of them sharing a birthday to be better than fifty-fifty—that is, for it to be more likely than not that any two share a birthday?